Convolution via Binding and Dot-Product in Vector Symbolic Architectures

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1 Introduction

I just briefly summarize how Vector Symbolic Architectures (VSAs) can approximately express convolution. Following Frady et al. [1], convolution can be reformulated within the VSA framework as a combination of binding operations and dot products.

2 Convolution

$$
\begin{aligned}
(f \circledast g)(u) &= \int f(r)\ g(u-r)\ \mathrm{d}r && (1)\\
&= \int f(r)\ \left(\int g(s)\ \delta(s-u+r)\ \mathrm{d}s \right)\ \mathrm{d}r && (2)\\
&= \int \int f(r)\ g(s)\ \delta(s+r-u)\ \mathrm{d}s\ \mathrm{d}r && (3)
\end{aligned}
$$

where u, r, s are indices (for example, positions in an image or a kernel).
In discrete space,

$$
(f \circledast g)(u) = \sum_r \sum_s f(r)\ g(s) \text{ where } u = r + s \tag{4}
$$

3 Vector Function Architecture (VFA)

3.1 Binding & Dot-product

Denote

- $\odot$ as binding operation (usually implemented using element-wise multiplication).
- $\cdot$ as dot-product.

3.2 Fractional Power Encoding (FPE) Vectors

Encoding $z(r) : \mathbb{N} \to \mathbb{C}^n$ is defined as

$$z(r)_j = e^{i\phi_j r} \tag{5}$$

where component j has a random phase ϕ_j.

Dot-product of two FPE vectors

$$z(r_1) \cdot z(r_2) = \sum_j^n e^{j\phi_j r_1} \ \overline{e^{j\phi_j r_2}} = \sum_j^n e^{j\phi_j (r_1 - r_2)} \tag{6}$$

By the Law of Large Numbers, as $n \to \infty$

$$\frac{1}{n} z(r_1) \cdot z(r_2) \to \mathbb{E}_{p(\phi)} \left[e^{j\phi(r_1 - r_2)} \right] = \int p(\phi) e^{j\phi(r_1 - r_2)} \approx K(r_1 - r_2) \tag{7}$$

so it can be also written as

$$z(r_1) \cdot z(r_2) \approx K(r_1 - r_2) \tag{8}$$

Binding of two PFE vectors

$$z(r_1) \odot z(r_2) = z(r_1 + r_2) \tag{9}$$

This property is trivial if $\odot$ is element-wise multiplication.
Note: $z(r)$ may also be generalized to a broader class of functions, provided they satisfy the two properties in equations (8) and (9).

3.3 Vector Representation of Functions

With VFA, a function $f(r)$ can be represented by a vector as

$$y_f = \sum_r f(r) \ z(r) \tag{10}$$

We can approximate element $f(s)$ by

$$f(s) \approx y_f \cdot z(s) \tag{11}$$

4 Convolution via Binding and Dot-Product

Given two vectors $f(r)$ and $g(s)$, binding of the two representation y_f and y_g can be written as

$$y_f \odot y_g = \sum_r f(r) z(r) \ \odot \ \sum_s g(s) z(s) \tag{12}$$

$$= \sum_r \sum_s f(r) g(s) \ (z(r) \odot z(s)) \tag{13}$$

$$= \sum_r \sum_s f(r) g(s) \ z(r + s) \tag{14}$$

From (4) and (11), convolution can be approximated as

$$(f \circledast g)(u) \approx (y_f \odot y_g) \cdot z(u) \tag{15}$$

References

[1] E. Paxon Frady, Denis Kleyko, Christopher J. Kymn, Bruno A. Olshausen, and Friedrich T. Sommer. Computing on functions using randomized vector representations, 2021.